## Effective Capacitance for the RC

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## Outline

- Background
- Prior Work
- A New Algorithm for Calculating the Effective Capacitance
- Experimental Results
- Conclusion


## Circuit Delay



$$
\text { Delay }_{A C}=\text { Delay }_{A B}+\text { Delay }_{B C}
$$

The circuit delay in VLSI circuits consists of two components:

1. the $50 \%$ propagation delay of the driving gates (known as the gate propagation delay)
2. the delay of electrical signals through the wires (known as the interconnect propagation delay)

## Gate Delay



Gate Delay $=f\left(T_{i n}, C_{\text {load }}\right)$

The gate load delay is a function of both input slew and the output load

## Library-based Delay Model



A pair of two-dimensional delay tables, one for providing the gate delay, the other for the output rise/fall time as a function of the effective load and the input transition time

## Second RC-p Model for Load

Using Taylor Expansion around $s=0$

$\hat{Y}_{i n}(s)=\left(C_{1}+C_{2}\right) s-R_{\pi} C_{2}^{2} s^{2}+R_{\pi}^{2} C_{2}^{3} s^{3}+\ldots .$.

$$
C_{1}=A_{1}-\frac{A_{2}{ }^{2}}{A_{3}} \quad R_{\pi}=-\frac{A_{3}{ }^{2}}{A_{2}{ }^{3}} \quad C_{2}=\frac{A_{2}{ }^{2}}{A_{3}}
$$

## Second RC-p Model (Cont’d)



$$
\text { Gate } \quad \text { Delay }=f\left(T_{i n}, C_{1}, R_{\pi}, C_{2}\right)
$$

Therefore, it is required to create a four-dimensional table to achieve high accuracy


This is however costly in terms of memory space and computational requirements

## Effective Capacitance Approach



The "Effective Capacitance" approach attempts to find a single capacitance value that can be replaced instead of the RC- $\pi$ load such that both circuits behave similar during transition

## Effective Capacitance (Cont’d)



$$
C_{e f f}=C_{1}+k C_{2}
$$

$0<k<1$

Because of the shielding effect of the interconnect resistance , the driver will only "see" a portion of the farend capacitance $C_{2}$


## Prior Work - Macys's Approach



Assumption: If two circuits have the same loads and output transition times, then their effective capacitance are the same.
In other words, the effective capacitance is only a function of the output transition time and the load

## Macys's Approach (Cont’d)



Normalized Effective Capacitance Function

$$
\begin{aligned}
& \alpha=\frac{C_{1}}{C_{1}+C_{2}} \\
& \beta=\frac{T_{\text {out }}}{R_{\pi} C_{2}}
\end{aligned} \quad \gamma=\frac{C_{\text {eff }}}{C_{1}+C_{2}} \quad 0 \leq \alpha \leq \gamma \leq 1
$$

## Macys's Approach (Cont’d)

1. Compute $\alpha$ from $C_{1}$ and $C_{2}$
2. Choose an initial value for $C_{\text {eff }}$
3. Compute $t_{\text {output }}$ for the given $C_{\text {eff }}$ and $T_{\text {in }}$
4. Compute $\beta$
5. Compute $\gamma$ from $\alpha$ and $\beta$
6. Find new $C_{\text {eff }}$
7. Go to step 3 until $C_{\text {eff }}$ converges

## Prior Work - Qian's Approach

Calculate the effective capacitance by equating the currents at the gate output by using:
(a) the driving-point admittance as the load
(b) using a single effective capacitance as the load

Average currents for both loads models are equated until the gate output voltage reaches the $50 \%$ threshold

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## A New Effective Capacitance Algorithm



$$
V_{M}(t)=\left\{\begin{array}{cc}
\frac{V_{d d}}{T_{R}}\left(t-B+A e^{-\alpha t} \operatorname{Cosh}(\omega t+\phi)\right) & 0 \leq t \leq T_{R} \\
\frac{V_{d d}}{T_{R}}\left(T_{R}+A^{\prime} e^{-\alpha t} \operatorname{Cosh}\left(\omega t+\dot{\phi}^{\prime}\right)\right) & T_{R}<t
\end{array}\right.
$$

## New Algorithm (Cont'd)



## Eff_Cap Equation

$$
C_{e f f}=\left(C_{1}+C_{2}\right) \frac{1-e^{-\alpha t} \frac{\operatorname{Cosh}(\omega t+\phi)}{\operatorname{Cosh}(\phi)}}{\left(1-e^{-\frac{t}{R_{d} C_{e f f}}}\right)}
$$

This is an Non-Linear Iterative Equation

A good initial value for $C_{\text {eff }}$ can speed up the procedure to find the answer


## Iterative Procedure to Calculate $\mathbf{C}_{\text {eff }}$

1. Start with the initial guess for $C_{\text {eff }}$
2. Obtain $\mathrm{t}_{0-50 \%}$ based on values of $\mathrm{C}_{\text {eff }}$ and $\mathrm{T}_{\mathrm{R}}$
3. Obtain $\mathbf{R}_{\mathrm{d}}$ based on values of $\mathrm{C}_{\text {eff }}$ and $\mathrm{T}_{\mathrm{R}}$
4. Compute a new value of $\mathrm{C}_{\text {eff }}$ from the Eff_Cap equation
5. Find new $\mathrm{t}_{0-50 \%}$ based on the new $\mathrm{C}_{\text {eff }}$ and given TR
6. Compare the values of $\mathrm{t}_{0-50 \%}$ from step 5
7. If not within acceptable tolerance, then return to step 3 until $\mathrm{t}_{0-50 \%}$ converges
8. Report $\mathrm{t}_{50 \%}$ propagation delay and $\mathrm{t}_{0-80 \%}$ from the table

## Extension to Complex Gates

To extend the previous algorithm to complex gates, we only need to compute the value of $R_{d}$


The gate output driver resistance changes as a function of the applied input waveforms

## Complex Gates (Cont’d)

First Order (simple) approximation:


Due to the body effect, this value is over-estimated

## Extension to Complex Gates,Cont'd

Our approach (using body effect coefficients):

$\mathrm{K}_{\mathrm{i}}$ is set to 1 if the corresponding input is not switching; Read
$\mathrm{K}_{\mathrm{i}}$ 's from a lookup table if the correspondent input is switching

## Experimental Results

| $\begin{gathered} \text { Inverter } \\ \text { Size } \\ (\mathrm{Wp} / \mathrm{Wn}) \\ \mathrm{mm} \end{gathered}$ | $\mathrm{C}_{1}(\mathrm{pF}) / \mathrm{R}_{\mathrm{p}}(\mathrm{W}) / \mathrm{C}_{2}(\mathrm{pF})$ | HSPIC <br> E 50\% <br> delay <br> (pS) | $\begin{gathered} \text { Estimated } \\ 50 \% \\ \text { delay } \\ \text { (pS) } \end{gathered}$ | Error | $\begin{gathered} \text { HSPICE } \\ 80 \% \\ \text { delay } \\ \text { (pS) } \end{gathered}$ | $\begin{aligned} & \text { Estimated } \\ & 80 \% \\ & \text { delay (pS) } \end{aligned}$ | Error | $\begin{gathered} \text { Number } \\ \text { of } \\ \text { Iteration } \\ \mathrm{s} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10/5 | 0.05/410/0.15 | 66.1 | 69.0 | 4.5\% | 142.3 | 140.6 | $\begin{aligned} & 1.2 \\ & \% \end{aligned}$ | 3 |
| 40/20 | 0.1/290/0.25 | 39.3 | 41.0 | 4.3\% | 95.3 | 97.4 | $\begin{aligned} & 2.2 \\ & \% \end{aligned}$ | 3 |
| 40/20 | 0.5/810/0.7 | 74.0 | 76.4 | 3.2\% | 136.2 | 134.5 | $\begin{aligned} & 1.3 \\ & \% \end{aligned}$ | 2 |
| 30/15 | 0.4/1000/0.8 | 76.3 | 79.4 | 4.1 \% | 142.5 | 138.5 | $\begin{aligned} & 2.8 \\ & \% \end{aligned}$ | 1 |
| 100/50 | 0.9/300/1.4 | 62.7 | 65.1 | 3.8\% | 121.0 | 123.1 | $\begin{aligned} & 1.7 \\ & \% \end{aligned}$ | 2 |
| Avg. <br> Error | -- | ------ | ------- | 4.0\% | --- | ----- | $\begin{gathered} 1.8 \\ \% \end{gathered}$ | ----- |

## Experimental Results

| 3-input <br> NAND <br> $(\mathrm{Wp} / \mathrm{Wn})$ | $\mathrm{C}_{1}(\mathrm{pF}) / \mathrm{R}_{\mathrm{p}}(\mathrm{W}) / \mathrm{C}_{2}(\mathrm{pF})$ | HSPICE <br> $50 \%$ <br> delay | Estimated <br> $50 \%$ delay <br> $(\mathrm{pS})$ | Error | HSPICE <br> $80 \%$ delay <br> $(\mathrm{pS})$ | Estimated <br> $80 \%$ <br> delay $(\mathrm{ps})$ | Error | Numbe <br> r of <br> Iteratio |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $20 / 60$ | $0.4 / 1000 / 0.8$ | 34.7 p | 36.4 p | $4.9 \%$ | 42.1 p | 43.2 p | $2.6 \%$ | ns <br> 2 |
| $40 / 120$ | $0.5 / 510 / 1.2$ | 26.4 p | 27.1 p | $2.7 \%$ | 78.1 p | 79.5 p | $1.8 \%$ | 2 |
| Avg. Error | $-\ldots----$ | $-\ldots---$ | $-\ldots---$ | $3.6 \%$ | $\ldots----$ | ------ | $2.2 \%$ | $-\ldots----$ |

The topmost transistor in the stack is switching

| $\begin{gathered} \text { 3-input } \\ \text { NAND } \\ (\mathrm{Wp} / \mathrm{Wn}) \end{gathered}$ | $\mathrm{C}_{1}(\mathrm{pF}) / \mathrm{R}_{\mathrm{p}}(\mathrm{W}) / \mathrm{C}_{2}(\mathrm{pF})$ | $\begin{aligned} & \text { HSPICE } \\ & 50 \% \\ & \text { delay } \end{aligned}$ | Estimated 50\% delay (pS) | Error | HSPICE $80 \%$ delay (pS) | $\begin{aligned} & \text { Estimated } \\ & 80 \% \\ & \text { delay } \end{aligned}$ | Error | $\begin{aligned} & \text { Numbe } \\ & \mathrm{r} \text { of } \\ & \text { Iterat. } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20/60 | 0.4/1000/0.8 | 64.7 p | 67p | 3.6\% | 64.4p | 64.8p | 0.6\% | 3 |
| 40/120 | 0.5/510/1.2 | 54.1p | 55.5p | 2.6\% | 83.5p | 84.5p | 1.2\% | 2 |
| Avg. Error | ------- | ----- | ------- | 3.1\% | ------- | ---- | 0.9\% | ----- |

All three transistors in the stack are switching

