An Analytical Model for Predicting the Remaining Battery Capacity of Lithium-Ion Batteries

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ABSTRACT

Predicting the residual energy of the battery source that powers a portable electronic device is quite important in designing and employing an effective dynamic power management policy in the device. This paper presents a closed-form analytical model for predicting the remaining capacity of a lithium-ion battery. The proposed high-level model relies on online current and voltage measurements and, at the same time, correctly accounts for the temperature and cycle aging effects. The accuracy of the high-level model is validated by comparing our analytical model with the Dualfoil simulation results (under some simplifying assumptions), demonstrating 5% error between simulated and predicted data.

1. INTRODUCTION

The battery service lifetime of a mobile embedded system is a major concern for hardware/software designers. Attempts for extending the battery lifetime have traditionally focused on minimizing the power consumption of the circuits powered by these batteries. However, these circuit-oriented techniques are not sufficient because they tend to ignore some important characteristics of the battery source itself. Examples of salient features of a (rechargeable) secondary battery are dependencies of the remaining battery capacity on its discharge-rate, temperature, the charge recovery effect, and the cycle aging effect. A number of researchers have started focusing on the battery sources and adapting the circuit optimization techniques and power management strategies to exploit the specific characteristics of the battery as a power source. Prediction of a battery's remaining capacity is the basis for developing effective battery-aware power optimization and management techniques. Both battery temperature and cycle life of a secondary battery have a large impact on the battery behavior. As temperature increases, the full discharge capacity tends to increase, but the total cycle-life tends to shrink significantly. The capacities of commercial lithium-ion batteries fade by 10-40% during the first 450 cycles. Without knowledge of temperature and cycle life of a battery, it is impossible to get to an accurate prediction of the battery remaining capacity.

This paper presents a fast prediction model to estimate the remaining capacity of a Lithium-Ion battery. The proposed analytical model takes the cycle aging and temperature effects into account. However, it requires that the battery history (i.e., its cycle age and its temperature profile over the past cycles) as well as instantaneous battery output voltage and the average current are known. The model is accurate only for a fixed average current drawn by the battery load. If the average current value changes, the model equations will have to be re-evaluated.

2. PROPOSED ANALYTICAL MODEL

State of charge (SOC) is a widely used metric used to represent the remaining capacity of a battery in a charge/discharge cycle. It is defined as the ratio of the *remaining capacity* (RC) to the *full charge capacity* (FCC). However, due to the cycle aging phenomenon, using SOC alone may result in large errors because the FCC of a cycle-aged battery may be significantly less than the *design capacity* (DC), which in turn denotes the FCC of a newly manufactured battery. Considering the cycle aging effect, a new concept, called *state of health* (SOH), is defined as the ratio of the FCC of a cycle-aged battery to its DC.

Based on Electrochemical analysis (details are omitted to save space), the discharged capacity c of a battery is related to the output voltage v of the battery, its average current draw i, the battery temperature T, and cycle count n_c by this equation:

$$b_1(i,T)\cdot c^{b_2(i,T)} = 1-\exp\left(\frac{r(i,T,n_c,T')\cdot i-(VOC_{init}-v)}{\lambda}\right) \tag{2-1}$$

In the above equation, VOC_{init} denotes the initial open-circuit voltage of a fresh battery, and $b_1(i,T)$ and $b_2(i,T)$ are related to the concentration evolution inside the battery during the discharge process. In addition,

$$r(i,T,n_c,T') = r_0(i,T) + n_c\cdot\sum_{T'} P(T')\cdot k\exp\left(-\frac{e}{T'}+\gamma\right) \tag{2-2}$$

where $r_0(i,T)$ represents the effects of the ohmic over-potential and the surface over-potential during the battery discharge process whereas the second term represents the cycle-aging effect. $P(T')$ denote the probability that the temperature is T' in some cycle. All other parameters are constant coefficients, which are determined from experimental data by curve fitting. Using equation (2-1) and (2-2), we derive the following:

$$DC = \left\{\frac{1}{b_1}\left[1-\exp\left(\frac{r_0\cdot i-\Delta v_m}{\lambda}\right)\right]\right\}^{\frac{1}{b_2}} \tag{2-3}$$

$$SOH = \left\{\frac{1-\exp\left(\frac{r_n\cdot i-\Delta v_m}{\lambda}\right)}{1-\exp\left(\frac{r_0\cdot i-\Delta v_m}{\lambda}\right)}\right\}^{\frac{1}{b_2}} \tag{2-4}$$

where $r_0 = r(i,T,0,T')$, $r_n = r(i,T,n_c,T')$, $\Delta v = VOC_{init} - v$ and $\Delta v_m = VOC_{init} - v_{cut-off}$. Notice that we have used b_1 and b_2 as short notation for $b_1(i,T)$ and $b_2(i,T)$. Based on these equations, SOC can be related to SOH and DC as follows:

$$SOC = 1 - \frac{\left[\frac{1}{b_1} - \left(\frac{1}{b_1} - SOH^{b_2} \cdot DC^{b_2}\right)\exp\left(\frac{\Delta v_m - \Delta v}{\lambda}\right)\right]^{\frac{1}{b_2}}}{SOH \cdot DC} \tag{2-5}$$

Finally, RC is calculated as:

$$RC = SOC \cdot SOH \cdot DC. \tag{2-6}$$

3. MODEL VALIDATION

The Dualfoil battery simulator [1] is used to simulate a Bellcore's PLION battery [2] and thereby evaluate the proposed model. After private correspondence with the authors of the DUALFOIL, the code was modified to incorporate a capacity degradation mechanism and nonlinear temperature dependences for the electrolyte transport and kinetic properties. First, to determine model parameters, a wide range of battery working conditions were simulated. The maximum parameter prediction error was less than 6.4% whereas the average error was 3.5%. We considered three test cases.

Case 1) The battery was cycled to 1200 cycles at "1C" rate at 20°C. The SOC profiles of the 200th, 475th, 750th and 1025th cycles are compared with the predictions of the proposed model in Figure 1.

Case 2) The battery was cycled to 200 cycles at 20°C. The discharge current of each cycle was assumed to be uniformly distributed in the range of C/15 to 4C/3. Next the battery was discharged at C/3, 2C/3 and C, and at 0°C, 20°C and 40°C. The remaining capacity profiles were compared with those predicted by the proposed model in Figure 2. The max prediction error is 4.2%.

Case 3) The battery was cycled to 360 cycles at "1C" rate. The temperature of each cycle was assumed uniformly distributed in the range from 20°C to 40°C. Next the battery was discharged at C/15 and 1C at 20°C. The simulation results were compared with the predictions of the proposed model in Figure 3. The max prediction error is 4.9%.

Note that in generating the base data by using the Dualfoil program, we relied on a simplified capacity degradation mechanism (i.e., a linear relationship) and a rather simple temperature dependence model (i.e., the Arrhenius relationship) for transport and kinetic properties of the electrolyte. Real batteries may follow more complicated relationships, and hence, the model prediction error may be higher.

REFERENCES

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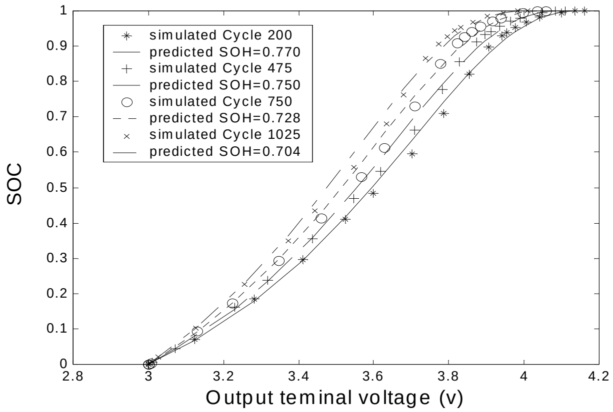

Figure 1. SOC traces for Test case 1.

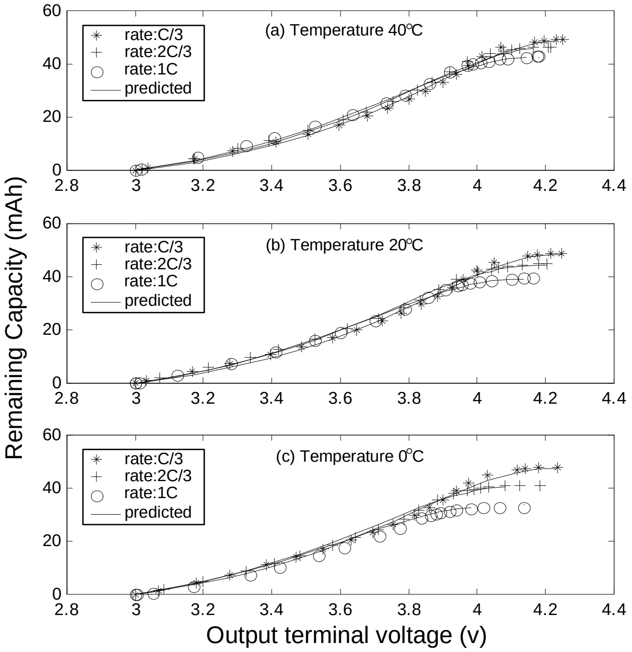

Figure 2. Remaining capacity traces for Test case 2.

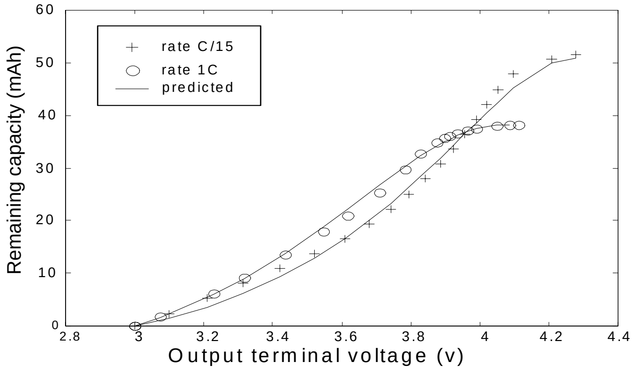

Figure 3. Remaining capacity traces for Test case 3.