

# Designing the Optimal Pricing Policy for Aggregators in the Smart Grid

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**Abstract**—The real-time pricing policy can incentivize the electricity users to dynamically change or shift their electricity consumption, thereby improving reliability of the grid. In the smart grid infrastructure, aggregators between the electricity suppliers and users control the users' electricity consumption by dynamically setting electricity price. This work aims at maximizing the overall profit of an aggregator in a billing period by designing a real-time pricing policy. The aggregator pre-announces a pricing policy for an entire billing period, then in each time interval of the billing period, the electricity users (i.e., both residential and EV users) try to maximize their own utility functions based on the pricing model in the current time interval and the awareness of the other users' behaviors. We first formulate a nested two-stage game between the aggregator and the users for each time interval in a billing period, in which the subgame perfect equilibrium can be found. Then, based on backward induction, a dynamic programming algorithm is presented to derive the optimal real-time pricing policy for maximizing the aggregator's overall profit. Different from other works, a battery energy storage system (BESS) is integrated with the aggregator to buffer the mismatch between supply and demand and to improve reliability of the grid. More importantly, this work derives the optimal pricing policy for an aggregator from a global point of view, taking into account the BESS energy state variation in a billing period. Simulation results show that the optimal pricing policy can achieve up to 24.3% improvement on the aggregator's overall profit.

**Keywords**—*smart grid; aggregator; real-time pricing; electric vehicle (EV); vehicle-to-grid (V2G)*

## I. INTRODUCTION

The increasing demands for energy resources all around the world as well as the growing public concern over the environmental effects of fossil fuels have sparked great interest in renewable energy. Electric vehicles (EVs), which use electric motors for propulsion, differ from fossil fuel powered vehicles in that the electricity they consume can be generated from a wide range of energy resources, including fossil fuels, nuclear power and renewable energy such as wind energy, solar energy and tidal energy. Therefore, switching from fossil fuel powered vehicles to EVs can be a promising solution to the energy crisis and environmental pollution [1].

Moreover, as most of the vehicles are parked on an average of 96% of the time [2], the concept of vehicle-to-grid (V2G) was discussed, where the electrical energy

storage ability of EV batteries is explored for frequency regulation, load balancing, etc [2][3][4]. Shimizu et al. proposed a V2G model considering EV customers' convenience and the state-of-charge control method by local control centers to suppress the grid frequency fluctuation [5]. Han et al. designed an aggregator for V2G frequency regulation service utilizing active discharge of EVs with the consideration of energy constraint of EV batteries [6]. Saad et al. formulated a non-cooperative game between EV groups (an EV group can, for example, represent a parking lot), which seek to sell part of their stored energy in a power market for maximizing a utility function capturing the tradeoff between the economical benefits from energy trading and the associated costs [7].

As a result of the growing demand for green energy technology, integration of EV charging/discharging and renewable energy scheduling into the grid can be mutually beneficial. Li et al. explored the controllable nature of EV charging to accommodate the intermittent wind energy with the aim of optimal electricity cost, replacing fossil fuel generation by wind energy, reducing ancillary service by controlled EV charging, and improving the quality of electricity service [8]. Wu et al. addressed the problem of integrating wind power into the grid by smart scheduling of end users' energy consumption using a game-theoretic pricing algorithm, considering both residential users and EV users, both shiftable loads and non-shiftable loads [9]. Mets et al. presented a distributed algorithm to increase the usage of wind energy while minimizing imbalance costs and the disutility experienced by consumers [10].

Traditional power grids are usually utilized to deliver electricity from central generators to a large number of users. On the contrary, in this paper we consider the smart grid infrastructure, where an automated and distributed energy delivery network is established between the electricity suppliers and users [11]. Between the electricity suppliers and users, aggregators are incorporated to reduce the amount of computation and communication overheads associated with the direct interaction between them. An aggregator coordinates the electricity consumption of (a group of) users by setting the electricity price in response to the imbalance between supply (the energy purchased from suppliers) and demand (the energy consumption of all users associated with the aggregator). For better reliability of the smart grid system, the aggregator employs a battery energy storage system (BESS) to buffer the mismatch between supply and demand. Each user (a residential user or an EV

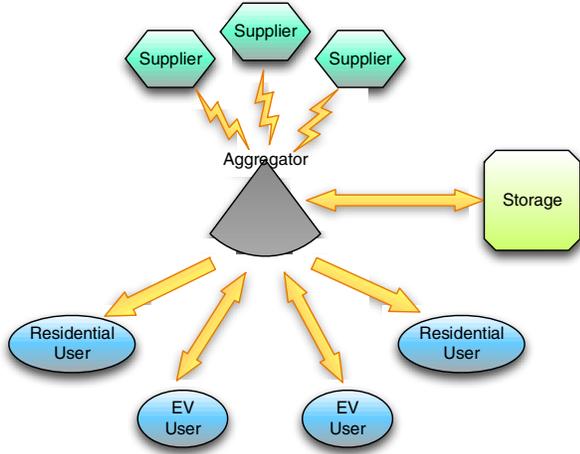


Figure 1. The System Structure.

user) is equipped with a software agent that schedules the user's energy consumption to pursue its best interest. The system structure is shown in Fig. 1.

We aim at maximizing the overall profit of an aggregator in a billing period by designing a real-time pricing policy. The aggregator pre-announces a pricing policy for an entire billing period, then in each time interval of the billing period, the electricity users (both residential and EV users) try to maximize their own utility functions based on the pricing model in the current time interval and the awareness of the other users' behaviors. First, we formulate a nested two-stage game between the aggregator and the users for each time interval in the billing period. The aggregator is the leader and the users are the followers. In the first stage of the game, the aggregator provides a pricing model (including the price coefficient and the amount of energy purchased from the suppliers) for this time interval. In the second stage of the game, the users (both residential and EV users) maximize their own utility functions based on the current pricing model and the awareness of the other users. We find the unique Nash equilibrium in the second stage of the game, which is the subgame perfect equilibrium in the nested two-stage game.

Then, we look back at the original problem: how the aggregator determines the pricing policy for an entire billing period such that its overall profit can be maximized. We use a dynamic programming algorithm to derive the optimal real-time pricing policy for maximizing the aggregator's overall profit, based on backward induction. Different from other works, we integrate a BESS with the aggregator to buffer the mismatch between supply and demand. More importantly, we derive the optimal pricing policy for an aggregator from a global point of view, taking into account the BESS energy state variation in a billing period.

We now proceed by discussing the system models including the pricing model, the overall profit of the aggregator in a billing period, the BESS stored energy, and the utility functions of users. We then propose the formulation of a nested two-stage game for each time

interval of a billing period. We derive the optimal pricing policy for maximizing the aggregator's overall profit using a dynamic programming algorithm based on backward induction. Simulation results are given to justify the effectiveness of the optimization procedure, followed by conclusions.

## II. SYSTEM MODEL

We present the system model in this section. Let us consider a smart grid infrastructure as shown in Fig. 1 and focus on the profit maximization of the aggregator. Between the electricity suppliers and users, the aggregator plays an important role in the system. It purchases electricity from suppliers and sells electricity to users. The aggregator can coordinate the electricity consumption of the users by setting electricity price in response to the imbalance between supply (the energy purchased from suppliers) and demand (the energy consumption of all users). For better reliability of the smart grid system, the aggregator employs a BESS to buffer the mismatch between supply and demand. The users can be categorized into two types: the residential users and the EV users. The residential users can only buy electricity from the aggregator, whereas the EV users can both buy and sell back electricity. The users determine their energy consumption according to the real-time electricity price set by the aggregator.

### A. Aggregator

We divide the entire billing period  $T$  (e.g., one day) into  $K$  discrete time intervals, each with length of  $\Delta T = T/K$ . Each time interval is indexed by an integer valued time index  $k$ . The aggregator purchases a total amount of  $D_k^0$  electrical energy from suppliers at the price of  $p_k^0$  in time interval  $k$ . The aggregator determines  $D_k^0$  for the entire billing period (for  $1 \leq k \leq K$ ), whereas  $p_k^0$  is set and pre-announced by the suppliers. Let  $D_k$  denote the total amount of energy consumption from all users in time interval  $k$ . In order to minimize the mismatch between  $D_k^0$  and  $D_k$ , the aggregator employs a pricing model as

$$p_k^s = p_{b1} + \alpha_k \cdot (D_k - D_k^0), \quad (1)$$

$$p_k^b = p_{b2} + \alpha_k \cdot (D_k - D_k^0), \quad (2)$$

where  $p_k^s$  is the price of selling energy to the users,  $p_k^b$  is the price of buying energy from EV users,  $p_{b1}$  and  $p_{b2}$  are the base prices, and  $\alpha_k$  is a positive coefficient. Deriving a pricing policy for an entire billing period is equivalent to determining the values of  $\alpha_k$  and  $D_k^0$  for  $1 \leq k \leq K$ . As can be observed in (1) and (2), when  $D_k > D_k^0$ ,  $p_k^s$  and  $p_k^b$  become higher than base prices, which discourages all users to consume more energy and encourages EV users to sell more energy back. On the other hand, when  $D_k < D_k^0$ , this pricing model encourages users to consume more energy and sell back less energy.

Let  $D_k^{neg} \leq 0$  denote the total amount of negative energy consumption from all users in time interval  $k$ . Then,

$|D_k^{neg}|$  is the total amount of energy the aggregator buys from EV users. The overall profit of the aggregator in a billing period is calculated as

$$profit = \sum_{k=1}^K (D_k - D_k^{neg}) \cdot p_k^s + D_k^{neg} \cdot p_k^b - D_k^0 \cdot p_k^0. \quad (3)$$

For better reliability of the smart grid system, the aggregator employs a BESS to buffer the mismatch between supply and demand. Assume that the BESS has a maximum energy capacity of  $E^{full}$ . Let  $E_k$  denote the energy stored in the BESS at the end of time interval  $k$ , then  $E_k$  can be derived by

$$E_k = E_{ini} + \sum_{t=1}^k (D_t^0 - D_t), \quad (4)$$

where  $E_{ini}$  denotes the energy stored in the BESS at the beginning of the billing period. Please note that  $0 \leq E_k \leq E^{full}$  should be satisfied for  $1 \leq k \leq K$ .

### B. Residential Users and EV Users

Assume there are a number of  $N$  users in the system. Let  $d_k^i$  denote the energy demand of user  $i$  in time interval  $k$ . If  $d_k^i \geq 0$ , the user  $i$  consumes energy purchased from the aggregator. If  $d_k^i < 0$ , the user  $i$  provides energy to the aggregator. The negative energy demand only applies to EV users. Then  $D_k$  defined in Section II.A can be calculated as

$$D_k = \sum_{i=1}^N d_k^i. \quad (5)$$

And  $D_k^{neg}$  defined previously can be obtained by

$$D_k^{neg} = \sum_{i=1}^N d_k^i \cdot \mathbf{I}[d_k^i < 0]. \quad (6)$$

where  $\mathbf{I}[\cdot]$  is an indicator function that equals to 1 if the input Boolean variable is true, and equals to 0 otherwise.

At any time interval, a residential user wants to minimize the cost for purchasing electricity from the aggregator and to maximize its own satisfaction level. As a combination of these two objectives, a residential user in a time interval maximizes a utility function in the following form:

$$UR_k^i = -b_k^i \cdot d_k^i{}^2 + c_k^i \cdot d_k^i - p_k^s(d_k^i, d_k^{-i}) \cdot d_k^i, \quad (7)$$

where  $d_k^{-i}$  denotes the energy demands of all the other users except for user  $i$ .  $p_k^s(d_k^i, d_k^{-i})$  is a function of both  $d_k^i$  and  $d_k^{-i}$  because it is an increasing function of the total demand  $D_k$  as given by (5).  $b_k^i$  and  $c_k^i$  in (7) are positive coefficients and they may be different for different time intervals and different users. The utility function is a concave function. In any time interval, a residential user maximizes its utility function  $UR_k^i$  by finding the most desirable  $d_k^i$  value, which depends on  $b_k^i$ ,  $c_k^i$ , and  $p_k^s(d_k^i, d_k^{-i})$ . Because residential

users can only buy electricity from the aggregator,  $d_k^i \geq 0$  is a constraint that must be satisfied when finding the most desirable  $d_k^i$  value.

On the other hand, an EV user can buy energy from the aggregator at the price of  $p_k^s$  as well as sell energy from its battery energy storage bank to the aggregator at the price of  $p_k^b$ . We define the following utility function for an EV user:

$$UEV_k^i = f_k^i \cdot \sqrt{g_k^i + d_k^i} - p \cdot d_k^i, \quad (8)$$

where

$$p = \begin{cases} p_k^s(d_k^i, d_k^{-i}) & \text{for } d_k^i \geq 0 \\ p_k^b(d_k^i, d_k^{-i}) & \text{for } d_k^i < 0 \end{cases} \quad (9)$$

$f_k^i$  and  $g_k^i$  are coefficients for user  $i$  in time interval  $k$ . The first term in (8) represents the energy level at the end of time interval  $k$ , and the second term in (8) represents the cost of charging or the revenue of discharging. Each EV user maximizes its utility function  $UEV_k^i$  by determining the most desirable  $d_k^i$  value according to the related coefficients and the behaviors of the other users.

### III. PROBLEM FORMULATION AND ALGORITHM

The aggregator tries to find the optimal real-time pricing policy (i.e.,  $\alpha_k$  and  $D_k^0$ ) in an entire billing period (for  $1 \leq k \leq K$ ), such that its overall profit in a billing period, given by (3), can be maximized. In order to derive the optimal pricing policy, we should understand the behaviors of the users in response to a pricing model in a time interval. Therefore, we formulate a nested two-stage game between the aggregator and the users for each time interval in a billing period. The game is described as follows.

For each time interval  $k$ :

- Stage I: The aggregator provides the total amount of energy purchased from suppliers (i.e.,  $D_k^0$ ) as well as the price coefficient (i.e.,  $\alpha_k$ ) to the users.
- Stage II: The users maximize their own utility functions by determining their own energy demands  $d_k^i$  (for  $1 \leq i \leq N$ ) with the information provided by the aggregator and the awareness of the other users' behaviors.

We can find the unique Nash equilibrium in Stage II, which is the subgame perfect equilibrium in the nested two-stage game. Then, we propose a dynamic programming algorithm to derive the optimal real-time pricing policy in a billing period. The algorithm is based on the backward induction method, which encapsulates the sequential rationality of decision makers and is used as a powerful technique to obtain the best strategies for the players in each stage of the nested game [12]. We will discuss the optimization procedure in detail as follows.

### A. Game Theoretic Optimization in Stage II

In Stage II of the nested game, considering time interval  $k$ , each user maximizes its own utility function (i.e.,  $UR_k^i$  or  $UEV_k^i$ ) by setting its energy demand  $d_k^i$  to a desirable value, according to  $\alpha_k$  and  $D_k^0$  (which has been provided by the aggregator in Stage I) and the awareness of the other users' behaviors. The total energy demand of all users, i.e.,  $D_k$ , affects the prices of energy, i.e.,  $p_k^s$  and  $p_k^b$ , through (1) and (2), thereby affecting the utility function of user  $i$  as given by (7) or (8). Therefore, the interactions between the users form a normal-form game, where all users take action simultaneously. We name this game the User Energy Demand Optimization (UEDO) game, which is a subgame of the nested two-stage game.

The Nash equilibrium of a normal-form game is the optimal strategy profile for all players in the sense that no player can find a better strategy (i.e., the  $d_k^i$  value) if he deviates from the current strategy unilaterally. In other words, no player (residential or EV user) has the incentive to leave its strategy in the Nash equilibrium. The Nash equilibrium of the UEDO game is the subgame perfect equilibrium of the nested two-stage game. Now we prove the existence and uniqueness of the Nash equilibrium in the UEDO game.

**Theorem 1:** The Nash equilibrium of the UEDO game exists and is unique.

*Proof:* According to the user utility functions, given by (7) and (8), we are essentially trying to maximize a strictly concave utility function for each player on a closed convex set. Therefore, from the first and third theorem in [13], the existence and uniqueness of the Nash equilibrium is proved.

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Algorithm 1: Find the Nash equilibrium of the UEDO game.

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**Input:** the total amount of energy purchased from suppliers i.e.,  $D_k^0$ , and the price coefficient i.e.,  $\alpha_k$ .

**Output:** the optimal energy demands of all users in time interval  $k$  i.e., the optimal  $d_k^i$  ( $1 \leq i \leq N$ ).

Initialize the energy demands of all users i.e.,  $d_k^i$  for  $1 \leq i \leq N$ .

**Do** the following procedure iteratively:

**For each**  $1 \leq i \leq N$ :

(1) Find the optimal  $d_k^i$  value for user  $i$  to maximize its utility function i.e., (7) for residential users or (8) for EV users, assuming that the energy demands of the other users are given.

(2) Update the  $d_k^i$  value for user  $i$ .

**End.**

**Until** the solution converges.

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The unique Nash equilibrium can be found using standard convex optimization technique [14], as described in Algorithm 1. Since  $\alpha_k$  and  $D_k^0$  are provided by the aggregator in Stage I, for different  $\alpha_k$  and  $D_k^0$  values, we can derive different Nash equilibrium by applying Algorithm 1.

We define two matrices  $\mathbf{M}_D^k$  and  $\mathbf{M}_D^{neg}$ , where each entry  $\mathbf{M}_D^k(\alpha_k, D_k^0)$  or  $\mathbf{M}_D^{neg}(\alpha_k, D_k^0)$  denotes the total energy demand  $D_k$  or the total negative energy demand  $D_k^{neg}$  in the Nash equilibrium obtained from Algorithm 1 with given  $\alpha_k$  and  $D_k^0$  values. These matrices are used to reduce the computation complexity later in the dynamic programming algorithm. Please note that for each time interval  $k$  ( $1 \leq k \leq K$ ), we build two matrices  $\mathbf{M}_D^k$  and  $\mathbf{M}_D^{neg}$ , since the coefficients  $b_k^i$ ,  $c_k^i$ ,  $f_k^i$  and  $g_k^i$  are different for different  $k$  value.

### B. Aggregator Overall Profit Optimization

Based on the analysis of user behaviors in response to a pricing model in one time interval, now we are ready to derive the optimal pricing policy in an entire billing period such that the aggregator's overall profit can be maximized. Consider a general problem of maximizing the profit made by the aggregator in the first  $k$  time intervals, such that by the end of time interval  $k$  the BESS stored energy is  $E'$ . We call this problem ( $E', k$ ) problem. If we solve ( $E', k = K$ ) problems with all possible  $E'$  values, we can find the solution to our original problem by picking one with the maximum profit in all ( $E', k = K$ ) problems. We find optimal substructure property of ( $E', k$ ) problem below, implying the applicability of dynamic programming algorithm.

**Optimal substructure property:** Suppose that ( $E', k$ ) problem has been optimally solved, and that in the solution the BESS stored energy is  $E''$  by the end of time interval  $k - 1$ . Then the optimal solution to the ( $E', k$ ) problem contains within it the optimal solution to the ( $E'', k - 1$ ) problem.

To solve the original optimal pricing policy problem, we maintain matrices **Prof**, **Alpha**, and  $\mathbf{M}_D^0$ . The entry  $\mathbf{Prof}(j, k)$  stores the maximum profit made by the aggregator in the first  $k$  time intervals of a billing period, when the energy stored in the BESS is  $j \cdot \Delta E$  by the end of time interval  $k$ , where  $\Delta E = E^{full}/M$  and  $j$  is an integer between 0 and  $M$ .  $\mathbf{Prof}(j, k)$  value will be obtained by solving ( $j \cdot \Delta E, k$ ) problem. Actually, we discretize the full range of the BESS energy state i.e.,  $[0, E^{full}]$  into a set of  $M + 1$  values i.e.,  $\{0, \Delta E, 2 \cdot \Delta E, \dots, M \cdot \Delta E\}$ . The aggregator's profit made in the first  $k$  time intervals can be calculated based on (3) as

$$prof_k = \sum_{t=1}^k (D_t - D_t^{neg}) \cdot p_t^s + D_t^{neg} \cdot p_t^b - D_t^0 \cdot p_t^0. \quad (10)$$

$\mathbf{Alpha}(j, k)$  and  $\mathbf{M}_D^0(j, k)$  store corresponding optimal  $\alpha_k$  and  $D_k^0$  values, respectively, that result in  $\mathbf{Prof}(j, k)$ .

In the dynamic programming algorithm, the matrix **Prof** will be filled up column by column (i.e., from  $k = 1$  to  $k = K$ ), and **Alpha** and  $\mathbf{M}_D^0$  will also be filled up together with **Prof**. When we finish the calculation of whole matrices, the last column of **Prof** will store the maximum profits made by the aggregator in the entire billing period when the BESS ends up with different energy levels. We

will pick the maximum one from  $\mathbf{Prof}(*, K)$  as the maximum overall profit of the aggregator, and the corresponding pricing policy will be obtained by tracing back [15].

Now let us look at how to derive the values of  $\mathbf{Prof}(j, k)$ ,  $\mathbf{Alpha}(j, k)$ , and  $\mathbf{M\_D}^0(j, k)$  i.e., how to solve  $(j \cdot \Delta E, k)$  problem. When we want to calculate  $\mathbf{Prof}(j, k)$ , we have known the values of  $\mathbf{Prof}(*, k - 1)$ . Therefore, we only need to find the optimal  $\alpha_k$  and  $D_k^0$  values in time interval  $k$ . We provide more details in the following. When  $\alpha_k$  and  $D_k^0$  are given, the profit made by the aggregator in the first  $k$  time intervals is calculated by

$$\mathbf{Prof}(j', k - 1) + (D_k - D_k^{neg}) \cdot p_k^s + D_k^{neg} \cdot p_k^b - D_k^0 \cdot p_k^0, \quad (11)$$

where  $D_k = \mathbf{M\_D}_k(\alpha_k, D_k^0)$ ;  $D_k^{neg} = \mathbf{M\_D}_k^{neg}(\alpha_k, D_k^0)$ ;  $p_k^s$  and  $p_k^b$  can be obtained by (1) and (2), respectively; and  $j'$  is determined by solving

$$(j - j') \cdot \Delta E = D_k^0 - D_k. \quad (12)$$

(12) implies the change of BESS stored energy from  $j' \cdot \Delta E$  to  $j \cdot \Delta E$  in the time interval  $k$  is due to the mismatch between the energy purchased from suppliers i.e.,  $D_k^0$  and the energy demand of all users i.e.,  $D_k$ . We need to find the optimal  $\alpha_k$  and  $D_k^0$  values, i.e.,  $\alpha_k^{opt}$  and  $D_k^{0,opt}$ , such that (11) is maximized. Then we set  $\mathbf{Prof}(j, k)$  as the maximum value of (11), and set  $\mathbf{Alpha}(j, k)$  and  $\mathbf{M\_D}^0(j, k)$  as  $\alpha_k^{opt}$  and  $D_k^{0,opt}$ , respectively. Algorithm 2 describes the dynamic programming algorithm for maximizing the aggregator's overall profit in a billing period.

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Algorithm 2: Maximize aggregator's overall profit.

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**Input:** matrices  $\mathbf{M\_D}_k$  ( $1 \leq k \leq K$ ), matrices  $\mathbf{M\_D}_k^{neg}$  ( $1 \leq k \leq K$ ), and the BESS stored energy at the beginning of the billing period i.e.,  $E_{ini}$ .

**Output:** the optimal pricing policy in the billing period i.e., the optimal  $\alpha_k$  and  $D_k^0$  for  $1 \leq k \leq K$ .

Initialize  $\mathbf{Prof}(*, 0)$  to  $-\infty$ .

Initialize  $\mathbf{Prof}(j = E_{ini}/\Delta E, 0)$  to 0.

**For**  $k$  from 1 to  $K$ :

**For**  $j$  from 0 to  $M$ :

        (1) Find  $\alpha_k^{opt}$  and  $D_k^{0,opt}$  values, such that (11) is maximized.

        (2) Set  $\mathbf{Prof}(j, k)$  as the maximum value of (11), and set  $\mathbf{Alpha}(j, k)$  and  $\mathbf{M\_D}^0(j, k)$  as  $\alpha_k^{opt}$  and  $D_k^{0,opt}$ , respectively.

**End.**

**End.**

Perform tracing back to find the optimal  $\alpha_k$  and  $D_k^0$  for  $1 \leq k \leq K$ .

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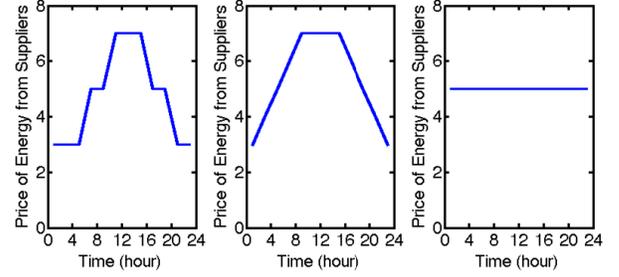


Figure 2. Three Sets of  $p_k^0$  Values.

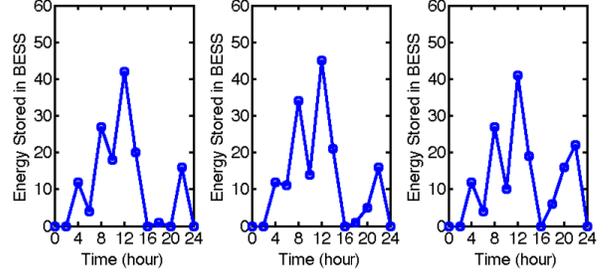


Figure 3. BESS Stored Energy as a Function of Time.

#### IV. EXPERIMENTAL RESULTS

In this section, we demonstrate experimental results of the optimal pricing policy for maximizing the aggregator's overall profit in a billing period. We compare the optimal pricing policy with a baseline pricing policy. The baseline pricing policy is different from the optimal pricing policy in that it finds the  $\alpha_k$  and  $D_k^0$  values for each time interval  $k$  such that the mismatch between  $D_k^0$  and  $D_k$  is zero and the profit made by the aggregator in each time interval is maximized.

In our simulation, we consider a group of 20 users associated with an aggregator. Among all the users, 10 of them are residential users and 10 of them are EV users. We consider a billing period of one day consisting of 12 time intervals, each with length of 2 hours. For the residential users,  $b_k^i$  is set as a randomized value between 2 and 3, and  $c_k^i$  is set as a randomized value between 175 and 225. For the EV users,  $f_k^i$  is set as a randomized value between 10 and 600,  $g_k^i$  is set as a randomized value between 35 and 200. The maximum energy capacity of the BESS (i.e.,  $E^{full}$ ) is 100 and the initial energy stored in BESS at the beginning of the billing period (i.e.,  $E_{ini}$ ) is 0. The base prices (i.e.,  $p_{b1}$  and  $p_{b2}$ ) are set as 8. The range of  $\alpha_k$  is set as  $[0.10, 0.30]$  and the range of  $D_k^0$  is set as  $[100, 800]$ . We use three sets of  $p_k^0$  values for  $1 \leq k \leq 12$  as shown in Fig. 2.

We derive the optimal pricing policy with different sets of  $p_k^0$  values using our proposed dynamic programming algorithm. As a result of the optimal pricing policy, we plot the BESS energy as a function of time for different sets of  $p_k^0$  values in Fig. 3. We can observe that the BESS energy starts from 0, reaches a peak value around the noon time, and finally goes back to 0. The mismatch between supply

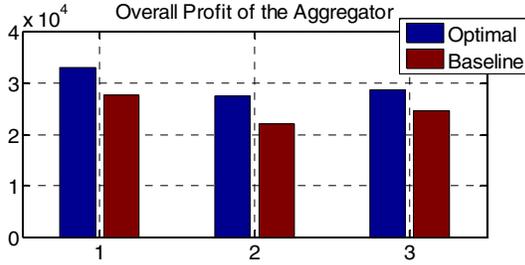


Figure 4. Overall Profit of the Aggregator from the Optimal Pricing Policy and the Baseline Pricing Policy.

(energy from suppliers) and demand (consumption of users) is totally buffered by the BESS.

The comparison between the optimal pricing policy and the baseline pricing policy is shown in Fig. 4. We can observe that the optimal pricing policy always achieves higher overall profit than the baseline pricing policy. The improvements achieved by the optimal pricing policy are 19.5%, 24.3%, and 16.4%, respectively, for the different sets of  $p_k^0$  values.

## V. CONCLUSION

In this work, we aim at maximizing the overall profit of an aggregator in a billing period by designing a real-time pricing policy. We first formulate a nested two-stage game between the aggregator and the users for each time interval in a billing period, in which the subgame perfect equilibrium can be found. Then, based on backward induction, we use a dynamic programming algorithm to derive the optimal real-time pricing policy for maximizing the aggregator's overall profit. Different from other works, we integrate a battery energy storage system (BESS) with the aggregator to buffer the mismatch between supply and demand and to improve reliability of the grid. Comparing with a baseline pricing policy, the optimal pricing policy can achieve a maximum of 24.3% improvement on the aggregator's overall profit.

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